OUNDLE

School

2016 Academic Scholarship

Mathematics

Paper II

Time Allowed: **2 hours**

Calculators may NOT be used for this paper

Instructions to candidates:

- You are not expected to have time to do all the questions.
- You may answer the questions in any order.
- Choose those questions which you think you can answer best.
- **Remember to show your working and clearly show the method you are using**.
- Answers should be given to 3 significant figures where appropriate.
- π may be taken as 3.14.
- **The number of marks for each question is show in square brackets**

1. George chooses a number between 1 and 100. He tells me that it has a remainder of 2 when divided by 5 and a remainder of 1 when divided by 3. List all of the possibilities for George's number.

[4]

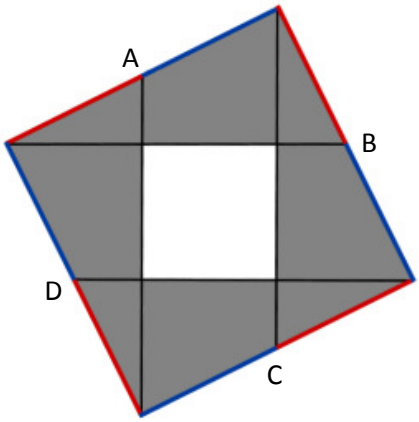
2. On Day 0, Daniel has £100 in his bank account. On day 1, he withdraws £1, on Day 2 he deposits £2, on Day 3 he withdraws £3, on day 4 he deposits £4 and continues in this way. On which day will Daniel have no money in his account?



[4]

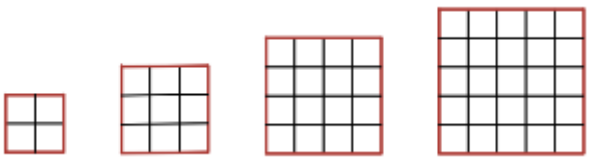
3. The perimeter of the large square below is 80m long, and points A, B, C and D (where the internal lines meet the edges) are the midpoints of each side.

   Find the area of the unshaded square.



[6]

4. I build several squares of different sizes made up of unit squares (squares of side 1) and I paint the perimeter of each in red paint.
   a) How many unit squares have exactly 1 edge painted in the square which is 4 x 4?
   b) How many would have only one edge painted in a 10 x 10 square?
   c) How many would have just one edge painted in an n x n square?



[6]

5. I have a rectangular piece of wood which is 156cm by 546cm and which I want to cut into square pieces of the same size. I want the squares to be as large as possible but without leaving any wood to waste.



What is the largest size of squares that I can cut?

[6]

6. The numbers *a* and *b* are whole numbers. If the value of *(a + b)* is odd, which of the following (A, B or C) is true about the expression:

$$(a + b)^2 + ab$$

(Give reasons for your choice)

A - $(a + b)^2 + ab$ is always even
B - $(a + b)^2 + ab$ is always odd
C - it depends on the values of *a* and *b*

[6]

7. The number 121 is an example of a palindromic number, that is, one that reads the same with the digits in reverse order. In the following infinite sequence of palindromic numbers, how many prime numbers will occur:

121, 12321, 1234321, 123454321…..?

[6]

8. At a particular party, every guest shakes hands with every other guest.
If there are 78 handshakes in total, how many guests attended the party?

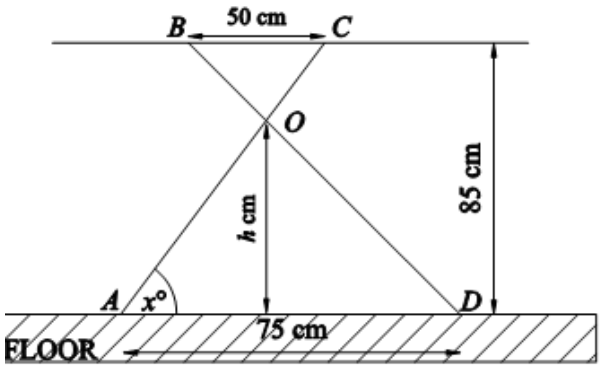


[6]

9. What is the area of a triangle with sides of length 9cm, 12cm and 15cm?

[6]

10. A table top rests on pairs of legs which intersect as shown in the diagram. The table top is 85cm above the floor and the ends of the legs are separated by 75cm at the floor and by 50cm at the table top as shown. Calculate *h*, the height of the intersection point of the table legs above the floor.



[6]

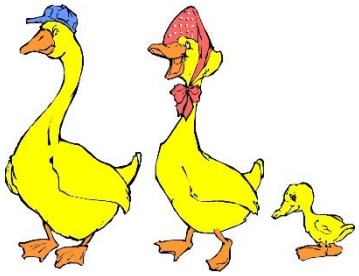
11. Demonstrate which of the following numbers are square numbers *stating clear reasons or working* for your decisions:

1728 1296 15876 128767

[8]

12. In a group of buffaloes and ducks the number of legs are 24 more than twice the number of heads. What is the number of buffaloes in the group?





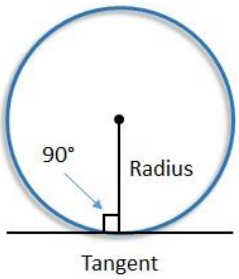
[8]

13. a) Isaac says that he has a set of 4 consecutive whole numbers, the sum of which is a multiple of 4. Can this ever be true? (You must clearly justify your answer)

b) Find all numbers p such that p, p+8 and p+16 are all prime.

[10]

14. A tangent to a circle is a straight line which just touches the circumference at one point. Where the radius of the circle meets the same point, the angle between the tangent and the radius is a right-angle as shown in the diagram.



Two circles A and B just touch each other at the point (4, 3). The line $y = \frac{3}{4}x$ passes through (4, 3) and is a tangent to both circles. If circles A and B both have radii of 5 units, find the coordinates of their centres. Note: you do not need compasses!

[8]

